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Modelling Rail Network Equilibria

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MODELLING RAIL NETWORK EQUILIBRIA

by

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FOREWORD

This report is the third in a series intended to develop an independent, cost effective ability to assess the cost and capacity implications of the changing nature of rail freight operations in Canada. It describes a first pass at developing a network equilibrium model of rail freight movements.

The model is applied to a hypothetical Prairie mainline and secondary mainline system very similar to the Canadian Pacific Saskatchewan network as it existed at the beginning of 1983. In addition, the implications of the following changes are modelled: a major yard expansion; an increase in the elasticity of demand for a major commodity shipped from the area; and an increase in mainline through traffic.

Any opinions which are expressed are those of the authors, and do not necessarily reflect those of the Canadian Transport Commission.

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The authors, however, assume full responsibility for the analysis and any errors that may be contained in this paper.

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1. The Network Equilibrium Problem and Its Computation

1.1 Introduction

The paper describes a first pass at developing an equilibrium model of rail freight movements. It is the third in a series of Canada Transport Commission research papers which are intended to develop an independent cost effective ability to assess the cost and capacity implications of the changing nature of rail freight operations in Canada.

Throughout North America there has been a movement to reorganize the rail system to reflect changes in technology and the competitive position of the mode. In Canada railroads must abide by federal legislative requirements such as the Railway Act and the Western Grain Transportation Act which stipulate that the railroads provide required services in an adequate and cost effective manner to shippers. The objective of this work is the development of a model which can easily and quickly estimate the implications of changes to the Canadian rail network.

The rail system is modelled as a set of separable cost functions associated with each of a large number of facilities called lines, yards and terminals. Each of these cost functions is assumed to be strictly convex in the total flow across that particular facility. These individual cost functions form the arcs of the network. Nodes of the network represent origins and destinations for freight traffic and a particular origin destination demand travels through the network over a series of facilities. The flow on a facility is just the sum of the demands which use that facility. All demands are elastic - as the price of the movement increases less is shipped.

Ultimately we seek to answer the following sorts of questions: given the present demands and flows in the network, how do these change if yard capacity at a particular point is increased or how does total demand change if a particular demand uses a different set of facilities or how do relative demands change given a change in demand elasticities? Thus we seek to compute an equilibrium given relatively small exogenous changes to the system.

An essential element of this model is accurate facility cost functions. Work in this area is ongoing at the Commission (see references 4 and 5).

In section 1 we describe the model and provide an algorithm for its solution. Section 2 provides the details for applying a FORTRAN code to solve the problem. We conclude Section 2 with a model of CP rail movements in Saskatchewan for 1984. This example is only intended as a prototype for a larger model of the western Canadian rail system.

1.2 Supply and Demand

The rail system is conceived as follows. Freight originates at a series of demand points and follows this typical path: first there is terminal work - a freight car is picked up at a shipper's siding and brought to the originating yard; it then moves over a series of lines and yards to its destination yard and then to the consignee where it is unloaded. Thus over its course from origin to destination a car travels through an originating terminal and yard, a series of yards and lines and a destination yard and terminal. (By way of definition, the destination terminal is the term used to describe the work of taking full cars from the destination yard to the consignee.)

However the car's trip does not stop at the consignee's warehouse. We must allow for the fact that the empty car may need to be transported to another point in the network. This point is discussed more fully at a later point in the paper.

Suppose we choose the unit of measurement to be cars per day and for the moment full and empty cars are not distinguished. Then cars move over a series of facilities, each having a cost function which depends on the total flow using that particular facility. Since cars are being moved from various points within the network simultaneously, the total flow on a particular facility depends upon which demands use the facility. And as facilities become more fully utilized they are subject to the congestion phenomena well known in the transportation literature. So given these considerations, suppose that the total cost of moving f_i cars per day on facility i is given by $F_i(f_i)$ where F_i is convex and increasing.

Within the network there are n O/D demands x_i, i=1,2,...,n measured in the units cars per day. For each O/D pair demand varies indirectly with the price charged - as the price rises demand for freight movements tends to decrease. For this reason we assume an inverse demand function $p_i^d(x_i)$ for O/D demand i, which is downward sloping and convex. Note also that $p_i^d(x_i)$ is a function only of x_i - it is not a function of any other demand.

There are m facilities which service demand requirements. These are of three types: terminals, yards and lines. Since routing is fixed, each O/D pair uses only a specified subset of all the facilities. To specify exactly what facilities the O/D pairs use we define a facility routing matrix R with elements r_{ij}. r_{ij} represents the addition to total flow on facility j induced by one unit of O/D demand i and is constant over the range of demand i.

If there were no problem with empty cars, we would let $r_{ij} = 1$ if O/D demand i used facility j. However if there are imbalances of demand within the network it will be necessary to move empty cars to points where they are needed. To see this consider the following simple example. Suppose there are two points in a network. Freight moves both ways along the line between yards A and B. Demand at A for B is 200 cars per day; demand for B at A is 180 cars per day. For all cars in the network it takes exactly one day to move from originating terminal to destination terminal and the process repeats itself daily. Now if there were no empty cars carried along the line from B to A there would have to be an infinite supply of cars available at A. The only situation which can persist in equilibrium is moving cars from A to B at a rate of 20 per day.

The fact that there are empties gives rise to two problems. First, a given set of demands for full freight car movements automatically gives rise to a set of demands to move empty cars to insure that there are not imbalances in the supply of empty cars. And secondly how should facility cost functions be adjusted?

To get around these problems we have taken the following approach. We have decided to leave facility cost functions as they are. Costs remain dependant on the total flow of cars - both empty and full - over the facility. Where we make the adjustment is in routing matrix elements r_{ij} . Suppose that demand i's full cars move across facility j and that empties attributable to demand i are routed back to demand i's origin by the same route. In this case rather than take $r_{ij} = 2$ we choose r_{ij} to be strictly less than 2, say in the neighbourhood of 1.8. Thus we make adjustment for differences in the facility costs for empties and fulls by adjusting the elements of the routing matrix.

In summary then, let f_i represent the total flow on facility i measured in cars per day where, again, we do not distinguish between empty and full cars. f_i is defined in terms of O/D demands x_i through the R matrix as follows:

$$f_i = \sum_{k=1}^{n} r_{kr} x_k \tag{1.1}$$

Equation 1.1 also makes clear another assumption and that is that a homogenous good is moved. Thus all flow movements are additive.

1.3 Equilibria

Given the assumptions in the previous section, the total cost of the railroad, T, is simply the sum of the individual cost functions:

$$T = \sum_{k=1}^{m} F_k$$

The marginal cost of moving x_i of O/D demand i is given by C_i where

$$C_i = \frac{\partial T}{\partial x_i} = \sum_{k=1}^{m} \frac{\partial F_k}{\partial x_i} = \sum_{k=1}^{m} \frac{\partial F_k}{\partial f_k} \frac{\partial f_k}{\partial x_i} \tag{1.2}$$

But, by definition, $\frac{\partial f_k}{\partial x_i} = r_{ik}$

Therefore,

$$C_i = \sum_{k=1}^{m} r_{ik} \frac{\partial F_k}{\partial f_k}$$

Thus to compute the marginal cost of O/D demand i we sum the marginal costs across all facilities which O/D demand i uses.

If the railroad were to price at marginal cost then we could define a marginal cost equilibrium as follows:

Definition 1

A *marginal cost equilibrium* is a vector of demands $x^M = (x_1^M, x_2^M, \ldots, x_n^M)$ such that $p_i(x_i^M) = C_i$ for all i.

While marginal cost pricing represents an efficient allocation of resources it may not correspond to actual railroad pricing schemes. Most railroads practise a form of average cost pricing where price is a fixed markup over average cost. This motivates the definition of an average cost equilibrium:

Definition 2

An *average cost equilibrium* is a vector of demands $\bar{x} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$ such that $p_i(\bar{x}_i) = \bar{C}_i$ for all i where $\bar{C}_i = \sum_{k=1}^{m} r_{ik} \bar{F}_k$ and $\bar{F}_k = \frac{F_k}{f_k}$

1.4 The Algorithm

Suppose we consider the computation of the marginal cost equilibrium. To show how this may be done using an optimization technique consider the following argument.

Define an excess demand vector $e(x) = p - C$ where $p = (p_1, p_2, \ldots, p_n)$ and $C = (C_1, C_2, \ldots, C_n)$. Also define the function $g = e^T e$. The equilibrium problem (EP) is to find an x such that $e(x) = 0$. Now suppose we found the minimum x^* of the function $g = e^T e$. Then clearly $e(x^*) = 0$ and $g(x^*) = 0$. Thus finding the minimum of g is equivalent to solving for the equilibrium.

The equivalent optimization problem (EOP) is

min $g(x) = e^T e$ for $x \geq 0$

Ignoring the non-negativity constraint this is an unconstrained optimization problem.

One solution to the problem is to use the method of steepest descent (line search along the opposite direction of the gradient). It is well known that this method converges. However in the application of this technique to the problem at hand convergence was extremely slow.

A second approach which, in our test problems, has shown remarkable efficiency, is to use a linear approximation to the facility cost functions at each iteration. A problem with this approach is that if the initial set of demands is too far away from the true equilibrium the algorithm may fail to converge.

2. Implementation

2.1 Data Requirements

To implement the model the following information is taken as exogenous:

1. the appropriate measure of flow;
2. the appropriate form of demand and supply functions as well as numerical estimates of their parameters; and
3. the routing matrix R.

We discuss each of these in turn.

Flow:

Suppose that cars per day is the appropriate measure of flow. This poses no problem for yard facilities since most yard models are based on cars per unit time. But for lines the natural unit of flow is trains per unit time. In a first pass at the problem we could assume a system wide equivalent flow such as 1 train per unit time = x_0 cars per unit time.

Demand and Facility Cost Functions:

The demand for rail transport is a derived demand. It depends critically on the demand for the goods which the railway moves. In this model we assume that demand is given exogenously. A convenient choice for this approach is the constant elasticity form:

$$D = A\left(\frac{C_0}{C}\right)^{\alpha}$$

where:

D is the demand measured in cars per day;

A is the current flow measured in cars per day;

C_0 is the cost of the current flow of A cars per day;

C is the endogenous cost of moving the equilibrium flow of D cars per day; and

α is the elasticity of demand.

Note that at $C = C_0$, $D = A$ and as C increases D decreases.

Thus in order to specify demand we need only two things: elasticity and current flow.

Given that railroads are susceptible to congestion phenomena, a reasonable choice for facility cost functions is the so called soft constraint type:

$$F_i(f_i) = \frac{a_i + b_i f_i}{1 - d_i f_i} \tag{2.1}$$

where F_i is the cost of moving f_i cars per day over facility i; and

a_i, b_i and d_i are facility parameters to be discussed below.

The graph of F_i is shown below in Figure 1. There are several things to note about F_i. First at $f_i = 0$, $F_i = a_i$. Thus a_i corresponds to the uncongested cost of the facility. a_i would include only the direct costs of the facility. For instance with lines we would include fuel, labour etc. - those things which are directly related to the operation of a train on a line.

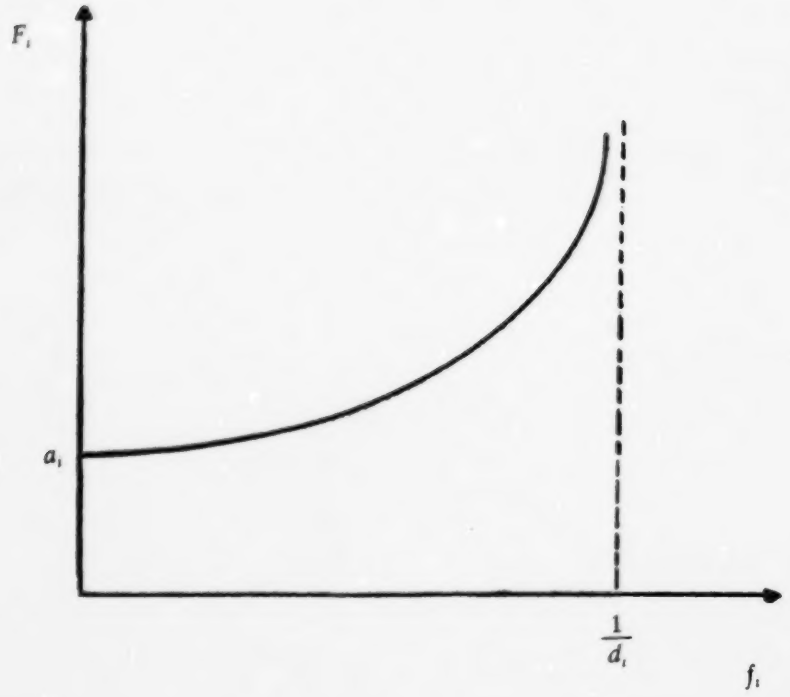

Figure 1: The graph of F_i

Also note that as f_i approaches $\frac{1}{d_i}$, F_i goes to positive infinity, since the denominator of the second term of the right hand side of equation 2.1 approaches zero. Thus $\frac{1}{d_i}$ is the capacity of the facility measured in cars per day.

The parameter b_i allows the slope of F_i to be adjusted. As b_i increases the slope of F_i increases.

Note also that this particular form of function can be reduced to the constant marginal cost form simply by letting d_i equal 0.

The R Matrix:

Each of a large number of demands will use only a small percentage of the facilities. Therefore only a small number of the elements of a row of R will be strictly positive. Thus rather than setting up R as a matrix and executing the algorithm as a series of matrix and vector multiplications using the full R matrix, it would be more appropriate to use an alternate data structure which takes advantage of the sparseness of R and does not cause a large number of multiplications by zero to be executed.

One possibility is a pointer system. Let Q_i be an ordered set which contains the column number of positive elements in row i. Associated with Q_i is the ordered set E_i which actually contains the positive elements of row i. Thus there is a one-to-one correspondence between the elements of Q_i and E_i.

Summary of Data Requirements:

The exogenous data requirements, then, are as follows:

1. Demand

   For each O/D pair we need α_i - the elasticity of demand, and A_i - the current flow in cars per day.

2. Facility Cost Function Parameters

For facilities we need a_i - the uncongested cost of using the facility, d_i - the inverse of the capacity of the facility, and B_i and b_i - the slope adjustment parameters.

3. The Routing Matrix, R

2.2 Software

RAILAR is a FORTRAN program which solves for the new equilibrium induced by the exogenous changes discussed above. RAILAR is designed to compile on Version 3.0 of Microsoft Fortran. It is available from the authors for a minimal charge.

The input data for RAILAR is set up in two files. One file contains demand information including the routing. The other contains facility cost information. Each of these has rather rigid formatting requirements which must be followed exactly if there are not to be errors at run time. The formatting for each of these is discussed in turn.

2.2.1 The Demand File

At execution RAILAR prompts the user for the name of the demand file so that any file with the appropriate data and format may be used. The file is constructed in three general sections. The first contains general demand information, the second contains the current routing and the third contains the new routing. Within and between sections there are dummy titles which must be included.

Line 1 contains N, the number of demands and is formatted I5.

The first section begins in line 2 and is exactly N lines long. These N lines consist of N records each with the following variables and formatting:

1. DID(I) (columns 1-5, format I5) is an integer demand identification number

2. DNAME(I) (columns 6-15, format A10) is a ten character name given to a demand
3. X(I) (columns 16-25, format F10.3) is a real number representing current level of demand for demand i
4. ELAST(I) (columns 26-30, format F5.3) is a real number representing the elasticity of demand i
5. MU(I) (columns 31-35, format I5) is an integer representing the number of facilities which demand i currently uses
6. MMU(I) (columns 36-40, format I5) is an integer representing the number of facilities which demand i uses in the new routing

The next line, line N+2, is a dummy read and contains the title for section 2 (columns 1-20, format A20)

Section 2 begins in line N+3 and is in two parts. The first part consists of specifying the routing and the second part consists of the elements of the R matrix.

The first part consists of 2N lines beginning in line N+3. For each demand there are two lines. The first contains a dummy read identifying the demand to which the following line pertains. The second line contains elements of RPOINT(I,J), the demand pointer. This line is formatted 10I5. Now it may be that demand i uses more than 10 facilities. If this is the case then the data for these additional facilities are specified on the following line for as many lines as are necessary.

The next line is a dummy read designed simply to separate the two parts of section 1. Its format is A25.

The second part of section 1 is constructed in the same manner as the first part. For each demand there are two lines. The first contains a dummy read identifying the demand to which the following line pertains. The second line contains elements of ROW(I,J), the elements of the R matrix. This line is formatted 10F5.3. Again, it may be that demand i uses more than 10 facilities. If this is the case then the data for these additional facilities are specified on the following line for as many lines as are necessary using 10F5.3.

The third section contains the new routing and is constructed the same way the second section is constructed.

Section 3 begins with a dummy read formatted A20. Any appropriate title may be used. Part 1 of Section 3 contains 2N lines. For each demand there are two lines. The first contains a dummy read identifying the demand to which the following line pertains. The second line contains elements of RRPOIN(I,J), the new demand pointer. This line is formatted 10I5. Now it may be that demand i uses more than 10 facilities. If this is the case then the data for these additional facilities are specified on the following line for as many lines as are necessary. Each of these additional lines is also formatted 10I5.

The rest of Part 2 is constructed in the same manner as Part 2 of Section 2. For each demand there are two lines. The first contains a dummy read identifying the demand to which the following line pertains. The second line contains elements of RROW(I,J), the elements of the R matrix. This line is formatted 10F5.3. Again, it may be that demand i uses more than 10 facilities. If this is the case then the data for these additional facilities are specified on the following line for as many lines as are necessary using 10F5.3.

2.2.2 The Cost File

At execution the user is prompted for the name of the cost file. This file is constructed in the same manner as the demand file

The first line contains M, the number of facilities and is formatted I5

The next M lines contain M records each with the following construction:

1. FACID(I) (columns 1-5, format I5) an ID number for facility i
2. FNAME(I) (columns 6-15, format A10) a 10 character name for facility i
3. A(I) (columns 16-25, format F10.3) the parameter a_i of the cost function for facility i
4. B(I) (columns 26-35, format F10.3) the parameter b_i of the cost function for facility i
5. D(I) (columns 36-45, format F10.3) the parameter d_i of the cost function for facility i

The next line is a dummy read with format A20.

The next section of the cost file is a set of 2M lines which contain the parameters of the new facility cost functions. There are two lines for each facility. On the first line there is a dummy read (format A5) specifying the name of the facility. The next line contains AA(I),BB(I), and DD(I) formatted 3F10.3

2.3 An Example

To test the algorithm we have applied the model to the Saskatchewan rail network. Saskatchewan was chosen for a number of reasons. The Research Branch of the CTC is currently investigating the rationalization of western grain movements by rail. Two Commission working papers suggest that there may be considerable economies by abandoning some low volume branch lines and by the increasing use of solid and unit grain trains (see references 1 and 6). The analysis in both of these papers is at best partial equilibrium. The rail network model in this paper is a first step in providing the means to extend these analyses to include equilibrium effects.

The network is shown in Figure 2. The mainline runs between Swift Current, Moose Jaw and Regina through to Brandon in the east and Medicine Hat in the west. A secondary mainline runs through Saskatoon to Edmonton in the west and ultimately to Winnepeg in the east. There are three other important sections of track: a line between Lanigan and Regina; a line from Moose Jaw south through North Portal to the United States; and a line from Estevan along the Estevan Subdivision through to Brandon.

The Saskatchewan economy is based primarily on agricultural and mining products. Grain, potash and coal are shipped by rail to export positions at Thunder Bay, Vancouver and, by CN, to Churchill in the north. Panel 1 of Table 1 presents the breakdown of full carloads originating in Saskatchewan for both CN and CP in 1983. Grain and mining products comprise 94% of the full carloads.

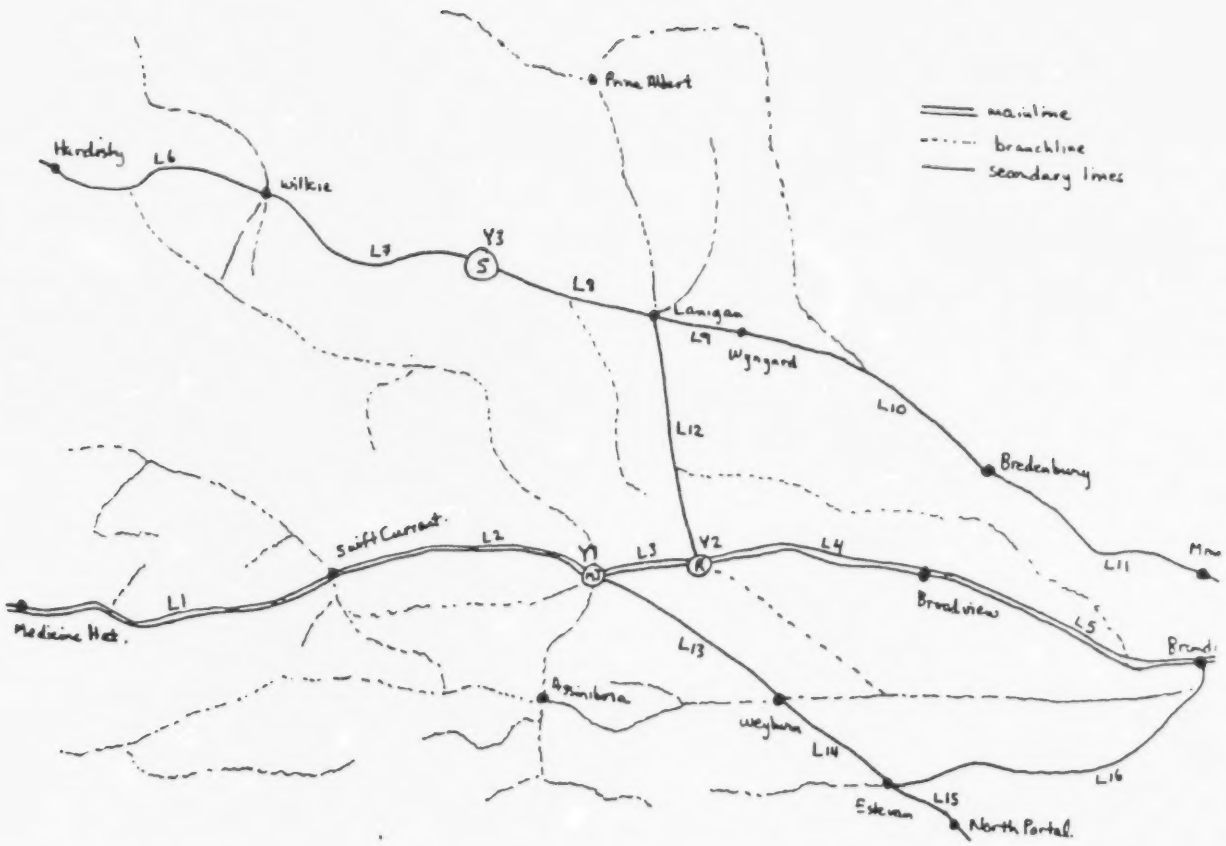

Figure 2: The CP Saskatchewan rail network

A directional breakdown of these flows is presented in Panel 2 of Table 1. Most of the grain moves east while mining products seem to be split. Table 2, which presents a directional breakdown of grain and mining movements between 1970 and 1983, provides some evidence that this relationship has persisted over time. In addition to these movements some potash

moves south through North Portal to the United States. Very little grain is transported through North Portal because these shipments do not receive the statutory grain rates.

Panel 3 of Table 1 shows the breakdown of the 42,106 cars having a destination in Saskatchewan. The majority of these are westbound. Panel 4 of Table 1 shows the breakdown of through traffic. There are a total of 114,710 eastbound cars and 81,706 westbound.

One can say some general things about this evidence. First, many more full cars leave Saskatchewan then arrive. Secondly, most of this traffic is grain and potash moving to Thunder Bay and Vancouver. And thirdly, the direction of full car movements is primarily east.

The rest of the data for this example have been derived from actual railroad data taken from the files of the CTC. Since some of this data is not intended for the public domain the numbers which follow have been altered.

For the purposes of this example we assume that there are 3 classes of products to move: grain, potash and "other". Table 3 presents a list of the individual demands in these classes as well as their associated elasticities and current demand. Assumptions about routing for each of these demands was made on the basis of the 1983 record of all CP traffic movements.

For line facilities, yards and originating stations provide natural terminal points for main and secondary mainline facilities. We also assume there to be terminal work at each of the originating stations. Facilities and their parameter estimates are provided in Table 4.

For all facilities we assume a_i to be zero. This is not a particularly strong assumption given the preliminary nature of the model. The line cost parameters were estimated in the following

manner. We assumed that each mainline facility had a blowup capacity of 40 trains per day and that secondary mainlines had one of 30 trains per day.[1] We assume the lower figure for secondary mains for reasons which include the nature of traffic control, fewer sidings and the operational characteristics allowable over the roadbed. To determine blowup in cars per day for main lines the figure of 40 is multiplied by the average length of train on that particular line. This number is just $\frac{1}{d_i}$

Given this estimate of d_i we next compute an estimate of b_i. Using the costing methodology developed in Spector(1987), a point on the cost curve for facility i is computed - say $(f_0 , F(f_o))$. Substituting this point and d_i into equation 2.1 gives an estimate of b_i. This same procedure gives parameter estimates for yards.

For terminals a slightly different approach was taken. We assumed d_i to be zero and estimated b_i to be the system wide average cost for moving a car on a local train.

To try the algorithm we looked at the following exogenous changes:

A. Yard capacity is expanded at Moose Jaw and Saskatoon from an assumed blowup capacity of 800 cars per day to 1200 cars per day.

B. The elasticity of potash movements is .8 rather than .2 and the change in A.

C. Mainline through movements are 50% higher and the changes in B and C.

The results of each of these changes are presented in Table 5. In scenario A total demand increases, although only marginally. In scenario B total demand doubles. However grain

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1 These blowup capacities may be determined from the physical layout of the facility using line and yard models developed by any number of authors. For examples see Petersen(1974) and Petersen(1977).

demand falls and potash demand increases substantially. This effect is magnified in scenario 3.

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